

# Study on Computerized Design Method for Cogeneration System Optimization

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## Abstract

To design the optimal co-generation system (CGS) by computer, there are two major difficulties to be solved.

One is to develop the evaluation function with sufficient accuracy, and the other is to develop the advanced algorithm which has a capability to handle with two different interdependent optimization problems such as determining system components and operation pattern of the selected components.

Regarding the evaluation function, a new mathematical model has been developed that calculates the amount of energy consumption in conjunction with the equipment database, by assigning integer value codes to specify equipment and real number to specify the hourly load factors from 0 to 1.

As for the optimization algorithm, it has been proven that the application of Estimation of Distribution Algorithm (EDA) makes outstanding solutions, as compared with the solution by usual Genetic Algorithm (GA).

## Introduction

Co-generation systems generate and supply electricity primarily by means of engine, turbine and fuel cell, and also retrieve and supply the waste heat produced through electricity generation. Therefore, they are expected to be widespread and contribute to boost total efficiency and save energy. Up to now, they have been primarily used for relatively large-scale systems such as, district heating and cooling, and recently down scaled CGS for home use with 1kW gas engine has been commercialized and the development of 1kW class CGSs with fuel cell is under going. Therefore, the spread speed of CGS is expected to accelerate in the future.

The most important point of the commissioning of CGSs is to grasp the energy demand of the facility where the system is to be installed, particularly in a case of the first new introduction.

Each system design must be customized for each customer, because energy demand varies with not only the type of customer's business such as; hotel, hospital, supermarket, but also each customer's business size. In today's system proposal, we have to satisfy the several different conditions: for example, suppressing the emission of NO<sub>x</sub>, SO<sub>x</sub>, in addition to the classical cost-minimizing requirement including initial cost and running cost.

In the future, moreover, the amount of CO<sub>2</sub> emissions may be added to the conditions that should be considered in the form of a tax levied on CO<sub>2</sub> emissions, emissions brokerage, and so on.

Since there are many conditions to be considered such as; design factors including demand pattern, the system configuration and the performance of component equipment, and evaluation criteria like cost and emission reduction, it is not easy to design the best CGS.

At present, a number of software programs which have been originally developed for the evaluation of the energy consumption in buildings are used in the design of CGSs. Nevertheless, sufficient studies have not been executed because of time restrictions, nor the study of parameters for appropriate designs consumes a great deal of time. This is essentially because of all design factor related to systems must be determined by man.

We think that the following goals will be attained by designing an optimal CGS automatically through the use of a computer.

Goals:

1. To drastically reduce the time needed for CGS design.
2. To perform a more cost effective and environmentally friendly CGS design.
3. To make optimal CGS design possible even for people with little experience.

It is the principle of the computerized optimization that the optimal design value will be obtained by the repetition of the generation of design values using the optimization algorithm and the assessment with an evaluation function, when applying the heuristic method. (Figure 1)

In the heuristic method, there are various optimization algorithms, such as the Simulated Annealing (SA), the TABU, and the GA. They are classified by the way how the design value is generated.

Among those algorithms, the GA can be adjusted to the problem which has many design variables simply by enumerating the design variables as genes. In the GA method, the parameters need not be adjusted with each problem, unlike with the cooling schedule of the SA or short term memory of the TABU.

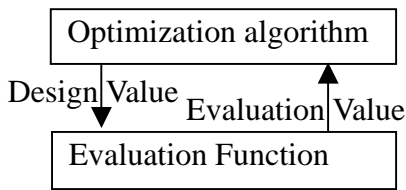


Figure 1. Concept of the Heuristic Method

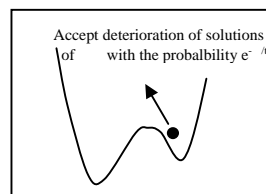


Figure 2. Concept of the SA

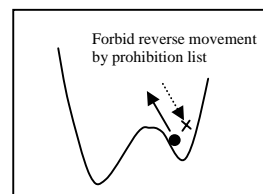


Figure 3. Concept of the TABU

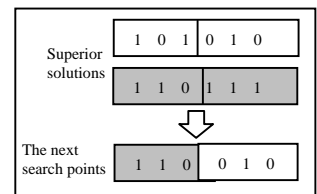


Figure 4. Concept of the GA

The optimization problem of the CGS is that the load factors of each piece of equipment and each unit of time are design variables, and that the function form of the evaluation function changes every time the equipment database changes.

It was decided to adopt an algorithm derived from the GA that could easily be applied to the problem of having many design variables, and that has a high tolerance for evaluation function changes.

In computerized automatic CGS design, both the evaluation function and the optimization algorithm have technical problems.

Problems:

1. The evaluation function should have fewer types of variables to be processed by the optimization algorithm. Additionally, it should be able to accurately calculate the amount of energy consumption, cost, CO<sub>2</sub>, and other environmental pollutants emission.
2. The optimization algorithm should perform two types of optimizations simultaneously: the deciding of the CGS equipment configuration and the operation pattern of the equipment, which are interdependent. In this thesis, the approach to and the results of problems 1 and 2, in order to obtain the stated goals 1 through 3, is introduced.

### Characteristics of the Developed Evaluation Function

#### (1) Limited Search Area

The number of CGS configuration patterns is so numerous, if uncommon systems are included, that it is difficult to make an evaluation function which completely covers any CGS. Granted that this could be achieved, the number of variables of the evaluation function would be so immense that there would be even greater difficulty in developing a corresponding optimization algorithm. Hence, a limit was put on the search area.

As an example, suppose the search area is a set of systems consisting of no more than two engines, two turbines, and two absorption chillers; and no more than one heat pump chiller, one waste heat recovery boiler, and one backup boiler. In this case, the system obtained through the optimization is the system of which the equipment G<sub>1</sub>, G<sub>2</sub>, T<sub>1</sub>, T<sub>2</sub>, AR<sub>1</sub>, AR<sub>2</sub>, EHP, B<sub>0</sub>, B<sub>1</sub> (as shown figure 5) are set as the evaluation value is minimum (or maximum). Here, it is permitted to set the equipment G<sub>1</sub>, ..., B<sub>1</sub> as "none".

As is shown, by determining the search area, the evaluation function can be developed, because it is the function which calculates the amount of energy consumption and cost of a determined CGS. Additionally, when the search area is broad enough, the optimal system will be chosen from among any CGS in practical use.

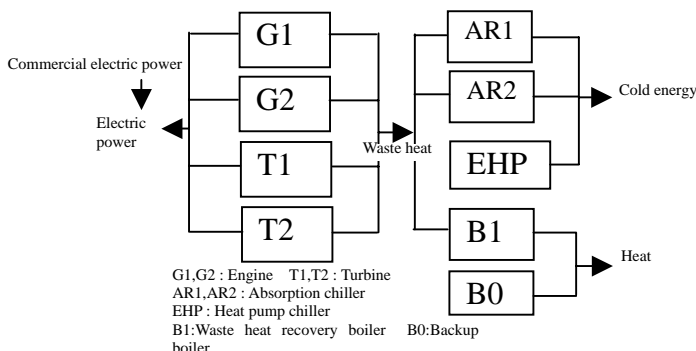


Figure 5. Examined cogeneration system configuration

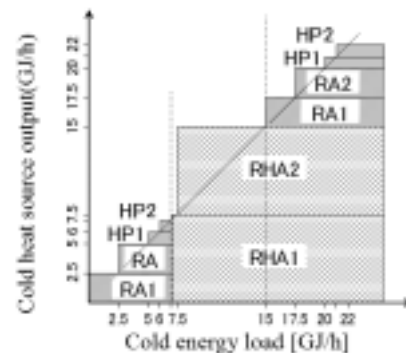


Figure 6. Operation Logic of Heat Source Equipment

#### (2) Design Variables

As design variables, at least the value indicating the equipment (G<sub>1</sub>, ..., B<sub>1</sub> in figure 5) and the value indicating the operation patterns of each piece of equipment are necessary.

The value indicating the equipment can be either the real number value of the rated output or the specific code number of the equipment. The value indicating the operation

pattern of the equipment can be the hourly load factor or the operation logic with which the operating equipment is specified for each heat load.(Figure 6)

We think the most versatile way is to adopt the code number of each piece of equipment as the design variable to decide the equipment, and the hourly load factor as the design variable to decide the operation pattern. So this method was chosen to develop the evaluation function.

### (3) Heat Calculation Method

Waste heat recovery equipment can only use specific form of waste heat: hot water, steam, exhaust gas, and so on, depending on the individual types of equipment. Therefore, the calculation had to be performed while taking the forms of waste heat into consideration. In many cases, waste heat takes the form of hot water, exhaust gas, low-pressure steam, or high-pressure steam. Due to this fact, heat calculation was performed for those four types of heat individually.

### (4) Equipment Performance Calculation

The performance of the equipment differs depending on the load factor, ambient temperature, and coolant temperature. Therefore, for each piece of equipment, we made mathematical formula models to calculate the necessary performance value based on conditions such as load factor.

## **CGS Energy Calculation Method**

### (1) Definition of Equipment

I will define the term “prime mover” as the equipment such as a gas engine, gas turbine and fuel cell, of which input is town gas and output is electricity and waste heat.

And define the term “waste heat recovery equipment” as the equipment such as a steam fired absorption chiller, hot water fired absorption chiller and heat exchanger, of which input is waste heat such as hot water, steam and exhaust gas, and output is heat for heating, cooling and hot water supply.

And define the term “auxiliary heat source equipment” as the equipment such as a gas fired absorption chillers, gas engine heat pump and electric heat pump, of which input is town gas and electric power, and output is heat for heating and cooling.

And define the term “heat storage equipment” as the equipment to store heat for heating, cooling and hot water supply such as hot water storage tank and cold water storage tank.

### (2) Energy Flow

The system configuration of the CGS which the developed evaluation function calculations is shown in Figure 7. The arrows that enter and exit the prime mover, boiler, waste heat recovery equipment, auxiliary heat source equipment, and heat storage equipment indicate the energy flow.

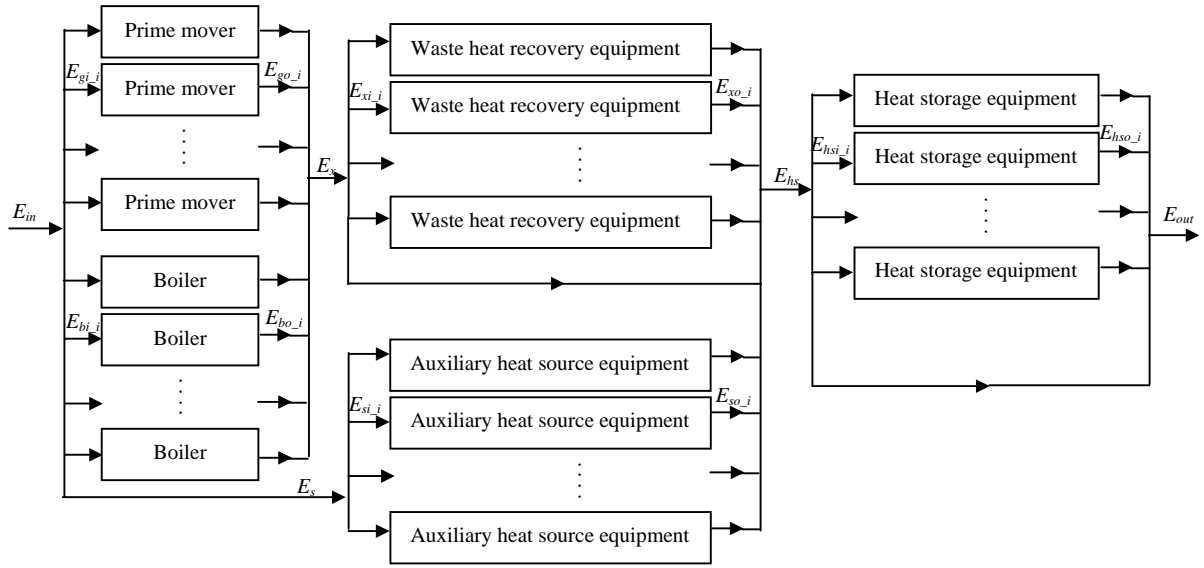


Figure 7. The System Configuration of the Calculation Objective CGS

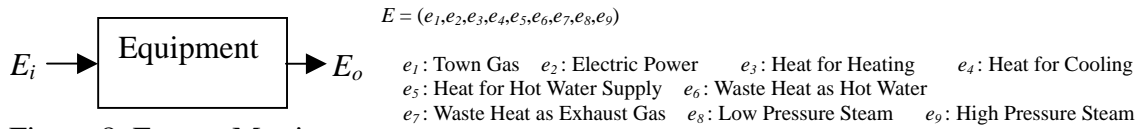


Figure 8. Energy Matrix

### (3) Energy Calculation Method

To simplify the calculation, the amount of energy per unit of time for town gas, electric power, heat for heating, heat for cooling, heat for hot water supply, waste heat as hot water, waste heat as exhaust gas, low pressure steam, and high pressure steam were taken and compiled as a single line matrix, shown in Figure 8.

The sums and differences of those energy amounts can be defined as a usual matrix, making the calculation easy when assembling or dispensing the waste heat of the equipment. It allows for the description of energy input and output of every piece of equipment in identical form, making it convenient for system calculation of various kinds of equipment.

In the calculation of each piece of equipment, both energy input  $E_i$  and energy output  $E_o$  were calculated. The values necessary for the calculation (input value) and the result of the calculation (output value) of each piece of equipment are shown in Table 1.

Among the input values, the equipment type and the load factor are automatically set by the optimization algorithm. The others are given as the calculation conditions for the optimization.

Within the evaluation function, the accurate energy calculation of the CGS was performed. However, it appeared to the optimization algorithm that the evaluate function carries out a simple performance of getting the input of the equipment type and the load factor and returning the evaluation value.

Table 1. Input and Output Variable of the equipment

Equipment	Input Variable	Output Variable
Prime mover	Equipment type, Load factor, Ambient temperature, Rated capacity, Power generation efficiency, Auxiliary power, Waste heat recovery efficiency	$E_i, E_o$
Boiler	Equipment type, Load factor, Rated capacity, Boiler efficiency	$E_i, E_o$
Waste heat recovery equipment	Equipment type, Load factor, Chilled water set temperature, Cooling water temperature, Auxiliary power, Rated COP, Rated capacity, Heat exchanger efficiency, Correction coefficient of energy Input.	$E_i, E_o$
Auxiliary heat source equipment	Equipment type, Load factor, Other variables depending on equipment.	$E_i, E_o$
Heat storage equipment	Equipment type, Load factor, Heat storage capacity	$E_i, E_o$

#### (4) Calculation Method for the Prime Mover

For the  $i^{\text{th}}$  prime mover, let  $E_{gi\_i}$  denote input and let  $E_{go\_i}$  denote output, at time  $h$ . Code number  $N_m$ , which specifies the equipment type, and load factor  $F_l$  are given as the design values, and ambient temperature  $T_a$  is given as the calculation condition.

The rated output  $W_0$  of the prime mover is obtained from code number  $N_m$ . When  $\eta_g$  stands for power generation efficiency of the prime mover, then

$$\eta_g = f(N_m, F_l, T_a) \quad \dots \text{Formula 1.}$$

When  $W_a$  stands for auxiliary power, then

$$W_a = f(N_m, F_l) \quad \dots \text{Formula 2.}$$

The amount of heat per unit time, which is in the form of hot water, exhaust gas, low pressure steam and high pressure steam, is denoted by  $O_{x1}, O_{x2}, O_{x3}, O_{x4}$ . Regarding waste heat  $O_{xi}$ , let  $\eta_{xi}$  stand for the waste heat recovery efficiency of the prime mover,

$$\eta_{xi} = f(N_m, F_l, T_a) \dots \text{Formula 3.}$$

In the computer program, the value of rated output  $W_0$ , and the function forms of formulas 1, 2, and 3 are obtained from the equipment performance database.

The input of the prime mover  $E_{fg\_i}$  is

$$E_{gi\_i} = (I, 0, 0, 0, 0, 0, 0, 0, 0) \quad \dots \text{Formula 4.}$$

$$I = \frac{F_l W_0}{\eta_g} \quad \dots \text{Formula 5.} \quad \text{Where } I \text{ denotes the town gas input per unit time.}$$

The output of the prime mover  $E_{xg\_i}$  is

$$E_{go\_i} = (0, O_e, 0, 0, 0, O_{x1}, O_{x2}, O_{x3}, O_{x4}) \quad \dots \text{Formula 6.}$$

Where  $O_e$  denotes electricity per unit time.

$$O_e = F_l W_0 - W_a \quad \dots \text{Formula 7.} \quad O_{xi} = I_g \eta_{xi} \quad \dots \text{Formula 8.}$$

#### (5) Calculation Method for the Boiler

Letting  $E_{bi\_i}$  denote input, and  $E_{bo\_i}$  denote output of the  $i^{\text{th}}$  boiler at time  $h$ , code number  $N_m$ , which specifies the equipment type, and the load factor  $F_l$  are given as design values. Rated output  $W_0$  of the boiler is obtained from code number  $N_m$ . Letting  $\eta$  stand for boiler efficiency,

$$\eta = f(N_m, F_l) \quad \dots \text{Formula 9.}$$

Boiler input  $E_{bi\_i}$  can be written as

$$E_{bi\_i} = (I, 0, 0, 0, 0, 0, 0, 0, 0) \quad \dots \text{Formula 10.}$$

$$I = \frac{F_l W_0}{\eta} \quad \dots \text{Formula 11.}$$

For boiler output  $E_{bo\_i}$ , when the boiler is a hot water boiler,

$$E_{bo\_i} = (0, 0, 0, 0, 0, O_{bo}, 0, 0, 0) \quad \dots \text{Formula 12.}$$

When the boiler is a high pressure steam boiler,

$$E_{bo\_i} = (0, 0, 0, 0, 0, 0, 0, 0, O_{bo}) \quad \dots \text{Formula 13.}$$

When the boiler is a low pressure steam boiler,

$$E_{bo\_i} = (0,0,0,0,0,0,0, O_{bo}, 0) \dots \text{Formula 14.}$$

Code number  $N_m$  decides the type of boiler.  $O_{bo}$  is obtained by the following formula:

$$O_{bo} = F_l W_0 \dots \text{Formula 15.}$$

## (6) Calculation Method of the Waste Heat Recovery Equipment

### 1. Absorption Chiller

As the mathematical description of the absorption chiller, a model was employed that was compatible with the TESS Library of the general energy system analyzing software, TRNSYS. TRNSYS was originally developed at the Wisconsin University Solar Energy Laboratory to analyze hot water supply and air conditioning systems using solar energy. It is presently used for the overall analysis of energy systems of buildings.

For the  $i^{\text{th}}$  waste heat recovery equipment, let  $E_{xi\_i}$  denote the input and  $E_{xo\_i}$  denote the output, at time  $h$ . Code number  $N_m$ , which specifies the types of equipment, and load factor  $F_l$  are given as design values. Chilled water set temperature  $T_{ch}$  and the cooling water temperature  $T_{co}$  are provided as the calculation conditions.

Auxiliary power of the waste heat recovery equipment  $Q_{AUX}$ , rated COP  $COP_0$ , and rated output  $W_0$  are obtained from code number  $N_m$ .

The input of the waste heat recovery equipment differs depending on the types of equipment which are obtained from code number  $N_m$ .

In the case of a low pressure steam fired absorption chiller,

$$E_{xi\_i} = (0,0,0,0,0,0,0, I, 0) \dots \text{Formula 16.}$$

In the case of a high pressure steam fired absorption chiller,

$$E_{xi\_i} = (0,0,0,0,0,0,0, 0, I) \dots \text{Formula 17.}$$

In the case of a hot fired absorption chiller,

$$E_{xi\_i} = (0,0,0,0,0, I, 0, 0, 0) \dots \text{Formula 18.}$$

In these formulas,

$$I = \eta I_0 \dots \text{Formula 19.}$$

The rated energy input is denoted by  $I_0$ ,

$$I_0 = \frac{W_0}{COP_0} \dots \text{Formula 20.}$$

$\eta$  is the correction coefficient of energy input.

$$\eta = f(F_l, T_{ch}, T_{co}) \dots \text{Formula 21.}$$

The output of the waste heat recovery equipment  $E_{xo\_i}$  is obtained by

$$E_{xo\_i} = (0,0,0,W,0,0,0,0,0) \dots \text{Formula 22.}$$

$$W = F_l W_0 \dots \text{Formula 23.}$$

## 2. Heat exchanger

This energy calculation method does not take into consideration the temperature of the heat transport medium. Due to this, the amount of exchanged heat cannot be calculated from the heat transfer performance. Hence, a simple model was adopted in which heat output is decided by multiplying heat input by  $\eta$ , taking the efficiency  $\eta$  as a constant.

For the  $i^{\text{th}}$  waste heat recovery equipment,  $E_{xi\_i}$  denotes input and  $E_{xo\_i}$  denotes output, at time  $h$ . Code number  $N_m$ , which specifies the types of equipment, and load factor  $F_l$  are given as the design values.

Rated output  $W_0$ , and the efficiency  $\eta$  are obtained from code number  $N_m$ . Input  $E_{xi\_i}$  is, when the heat exchanger is for hot water,

$$E_{xi\_i} = (0,0,0,0,0,I,0,0,0) \dots \text{Formula 24.}$$

$$\text{For low pressure steam, } E_{xi\_i} = (0,0,0,0,0,0,0,I,0)$$

$$\text{For high pressure steam, } E_{xi\_i} = (0,0,0,0,0,0,0,0,I)$$

The output  $E_{xo\_i}$  is when the heat exchanger is for heating,

$$E_{xo\_i} = (0,0,W,0,0,0,0,0,0) \dots \text{Formula 25.}$$

$$\text{For hot water, } E_{xo\_i} = (0,0,0,0,W,0,0,0,0)$$

In these formulas,  $I$  and  $W$  can be obtained from the following formulas:

$$I = \frac{W}{\eta} \quad \text{and} \quad W = F_l W_0 \dots \text{Formula 26.}$$

## (7) Calculation Method of the Auxiliary Heat Source Equipment

In the same manner as the prime mover, boiler, and waste heat recovery equipment, input  $E_{si\_i}$  and output  $E_{so\_i}$  are calculated using code number  $N_m$  and load factor  $F_l$ .

## (8) Calculation Method of the Heat Storage Equipment

For the  $i^{\text{th}}$  heat storage equipment, input is denoted by  $E_{hsi\_i}$ , and  $E_{hso\_i}$  denotes

output, at time  $h$ . Code number  $N_m$ , which specifies the type of equipment, and load factor  $F_{l_{h-1}}$  at time  $h-1$ , and load factor  $F_{l_h}$  at time  $h$  are given as the design values. From code number  $N_m$ , the performance of the equipment  $V_p$  is obtained.

Output  $E_{hso\_i}$  is, when the equipment is a hot water storage tank for hot water supply,

$$E_{hso\_i} = (0,0,0,0,W,0,0,0,0) \dots \text{Formula 27.}$$

In this formula,  $W = (F_{l_{h-1}} - F_{l_h})V_p + I - Q \dots \text{Formula 28.}$

$$Q = f(F_{l_{h-1}}) \dots \text{Formula 29.}$$

In these formulas,  $W$  and  $I$  have to meet these specifications:  $W \geq 0$ ,  $I \geq 0$  and  $W=0$  or  $I=0$ . Both  $W$  and  $I$  will be decided uniquely, using formula 28 and the above conditions.

Input  $E_{hsi\_i}$  is, if the equipment is a hot water storage tank for hot water supply,

$$E_{hsi\_i} = (0,0,0,0,I,0,0,0,0) \dots \text{Formula 30.}$$

If the equipment is for waste heat as hot water, then

$$E_{hsi\_i} = (0,0,0,0,0,I,0,0,0) \dots \text{Formula 31.}$$

#### (9) Calculation of the Entire System

The entire amount of energy input  $E_{in}$  of the CGS, at time  $h$ , is obtained by the following formula:

$$E_{in} = \sum E_{gi\_i} + \sum E_{bi\_i} + \sum E_{si\_i} \dots \text{Formula 32.}$$

The entire energy output  $E_{out}$  of the CGS is obtained from the following formulas:

$$E_{out} = \sum E_{hso\_i} + E_{hs} - \sum E_{hsi\_i} \dots \text{Formula 33.}$$

$$E_{hs} = \sum E_{xo\_i} + E_x - \sum E_{xi\_i} + \sum E_{so\_i} \dots \text{Formula 34.}$$

$$E_x = \sum E_{go\_i} + \sum E_{bo\_i} \dots \text{Formula 35.}$$

The minimum requirement for CGS design is that  $E_{out}$  meets the energy demand at all times.

### Utilization of the Evaluation Function without Optimization Algorithm

Although the evaluation function is for CGS optimal design software, it can also be used for evaluating a CGS that is manually designed.

At present, an optimization algorithm which searches for the optimal solution with this evaluation function is under development. So, We use the evaluation function for CGS

design with the program which sets the system configuration manually and the operation pattern according electric demand instead of the optimization algorithm.

### Technical Problems of the Optimization Algorithm

The developed evaluation function takes discontinuous values toward changes of equipment configuration and includes nonlinear equipment performance data. Also, since it has to decide equipment load factors for all time units, the number of design variables is innumerable. Therefore, We decided to develop an optimization algorithm based on the GA, which has no limitations in the forms of evaluation functions and can be applied to the problem that has innumerable design variables.

Even with the use of the GA, the optimization of the CGS is difficult, as it has two different kinds of design variables, the equipment configuration and the operation pattern, which are interdependent.

The basic principle of the GA is that the optimal solution is searched out as genes are exchanged among superior individuals. To work this mechanism, the individuals that result from genetic exchange of individuals must likewise be superior. However, this does not become true in CGS optimization.

For instance, as shown in Figure 9, superior individual 1 is a CGS consisting of a 50kW gas turbine that emits exhaust gas as waste heat and a 50kW absorption chiller that recycles exhaust gas as heat energy. The load factor at times 1, 2, and 3 are 100%, 90%, and 80% for the gas turbine, and 90%, 80%, and 70% for the absorption chiller.

Superior individual 2 is a CGS consisting of a 100kW gas engine that emits hot water as waste heat and a 100kW absorption chiller that recycles hot water as heat energy. The load factor at times 1, 2, and 3 are 50%, 40%, and 40% for the gas engine, and 40%, 40%, and 30% for the absorption chiller.

However, individual 3, which was generated by crossing over superior individuals 1 and 2, is a combination of a gas turbine that emits exhaust gas as waste heat and an absorption chiller that uses hot water as its energy input. It cannot recycle the waste heat, and the load factor is not optimal. This is because of:

1. Interdependence of the equipment types and the load factor.
2. Interdependence of the equipment.

So, we conceived to enable the genetic algorithm to take these interdependencies into consideration.

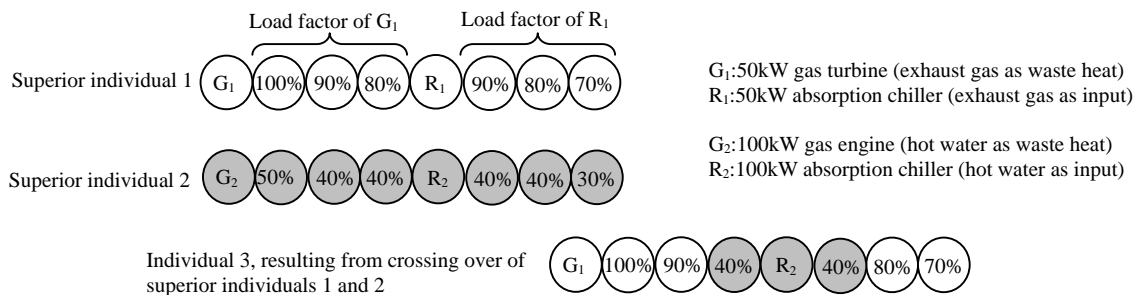


Figure9. Problems of Application of Genetic Algorithm

## Application of EDA

The Estimation of Distribution Algorithm (EDA) is an algorithm that searches for the optimal solution as it repeats the following processes:

1. It samples points with a high evaluation value from among the points that are searched.
2. It estimates the probability distribution of optimal solutions from the distribution of sampled points. In other words, it estimates the probability that set A includes the optimal solution; in this case,  $A \in F$ , and F is a set of searched points within a search space.
3. It determines the next search point according to the estimated distribution.

The EDA is an algorithm in which the crossing over and mutations in genetic algorithm are changed with the estimation of probability distribution of optimal solution and the occurrence of the next generation. It is also called the Probabilistic Model Building Genetic Algorithm. As was stated, in the optimization problems of a CGS, the interdependency of variables needs to be taken into consideration. Therefore, we conceived to apply the EDA, in which the principal component analysis is taken during the estimation process of the probability distribution of the optimal solution, to this optimization problem.

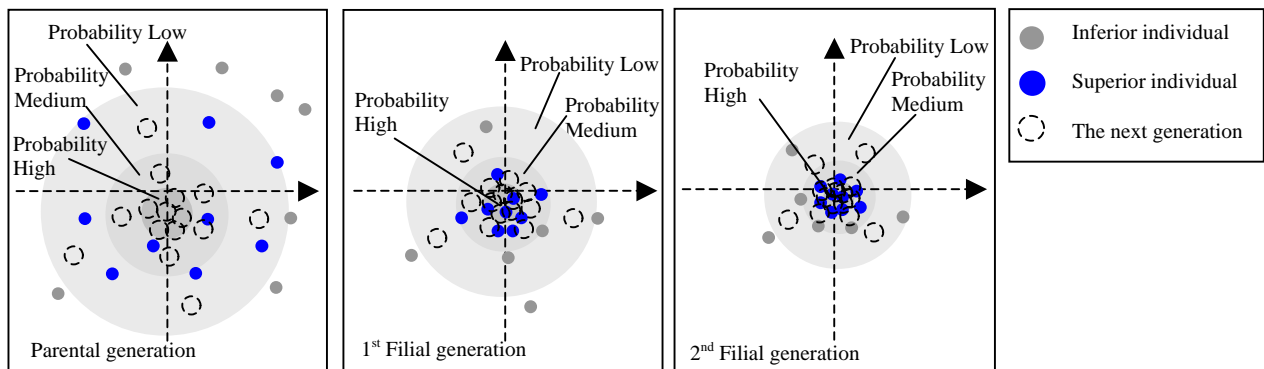


Figure 10. Solution Search by EDA

## About EDA

Below is an explanation of the principle of the EDA that we attempted to apply to the CGS optimization:

Let  $N_g$  stand for the number of individuals of one generation. Let  $N_d$  stand for the number of design variables. Let  $D_k$  stand for the gene of the  $k^{\text{th}}$  individual.

$D_k$  is a real number vector, and it is defined as follows;

$$D_k = (d_1, d_2, d_3, \dots, d_{N_d}) \quad \dots \text{Formula 36.}$$

$N_e$  represents the number of superior individuals taken from among  $N_g$ , and their genes  $D_k$  are placed as elements in matrix U.

$$U = \begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ D_{N_e} \end{bmatrix} \quad (U \text{ is a } N_e \times N_d \text{ matrix}) \dots \text{Formula 37.}$$

$U_{i,j}$  stands for the element of matrix U at row i, column j.  
Matrix T is made from matrix U, based on the following formula;

$$T_{i,j} = U_{i,j} - \frac{1}{N_e} \sum_k U_{k,j} \quad \dots \text{Formula 38.}$$

T is a matrix that consists of vectors resulting from translating  $D_1, D_2, D_3, \dots, D_{N_e}$  by their centroid  $\frac{1}{N_e} \sum_k D_k$ .

Next, consider obtaining matrix Y, which is the rotation of T in the principal direction. First, create matrix S from matrix T.

$$S = \frac{1}{n-1} T^T T \quad \dots \text{Formula 39.}$$

Next, obtain the eigen values  $\lambda_1, \lambda_2, \dots, \lambda_{nd}$  of matrix S. Then, obtain eigen vectors (column vectors)  $V_1, V_2, \dots, V_{Nd}$  corresponding to the obtained eigen values  $\lambda_1, \lambda_2, \dots, \lambda_{nd}$ , and rearrange  $V_i$  in descending order from  $\lambda_i$ , beginning with the greatest value. Place  $V_i$  and create matrix V.

$$V = [V_1 \ V_2 \ \dots \ V_{Nd}] \quad \dots \text{Formula 40}$$

Matrix Y is the product of matrices T and V.

$$Y = TV \quad \dots \text{Formula 41.}$$

From this Y, the individual of the next generation is created. First, obtain the average and variance of column j of Y.

$$\bar{Y}_j = \frac{1}{N_e} \sum_k Y_{k,j} \quad \dots \text{Formula 42.}$$

$$\bar{\sigma}_j^2 = \frac{1}{N_e} \sum_k (Y_{k,j} - \bar{Y}_j)^2 \quad \dots \text{Formula 43.}$$

Generate random numbers  $r_1, r_2, \dots, r_{N_e}$  by normal random numbers which have the average  $\bar{Y}_j$ , variance  $\bar{\sigma}_j^2$ . Vector  $R_j$  is defined as follows;

$$R_j = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_{Ne} \end{bmatrix} \dots \text{Formula 44.}$$

Create matrix  $Y_{offs}$  from  $R_j$ ;

$$Y_{offs} = [R_1 \quad R_2 \quad \dots \quad R_{Nd}] \dots \text{Formula 45.}$$

This  $Y_{offs}$  becomes the gene of the next generation which is translated and rotated. By the reverse manipulation of creating  $Y$  from  $U$ ,  $U_{offs}$ , the matrix of the genes of the next generation, can be obtained from  $Y_{offs}$ .

First, rotate matrix  $Y_{offs}$  by  $V^{-1}$ .

$$T_{offs} = Y_{offs} V^{-1} \dots \text{Formula 46.}$$

Next, create the matrix  $U_{offs}$  by translating  $T_{offs}$ .

$$U_{offs\_i,j} = T_{offs\_i,j} + \frac{1}{n} \sum_k U_{k,j} \dots \text{Formula 47.}$$

The gene of the  $k^{\text{th}}$  individual of the new generation,  $D_k$ , is expressed by the following formula:

$$D_k = (U_{offs\_k,1}, U_{offs\_k,2}, U_{offs\_k,3}, \dots, U_{offs\_k, Nd}) \dots \text{Formula 48.}$$

The above is the procedure to create the filial generation from the parental generation. By repeating this procedure, a more optimal design value can be obtained.

## Applying EDA in CGS Optimization

### (1) Conditions of Analysis

Three types of energy demand are taken into account as follows: electric power demand, heat demand, and cold energy demand. The energy demand of a day is assumed as shown in Figure 11. Also, the examined CGS configuration are set to be consisted with a generator, waste heat recovery boiler, absorption chiller, backup boiler, and heat pump chiller (EHP) (Figure 12). In this example, the design parameters to be searched by EDA are;

- 1 Rated electrical output of generator
- 2 Hourly load factor of generator
- 3 Rated output of waste heat recovery boiler
- 4 Hourly load factor of waste heat recovery boiler
- 5 Rated output of absorption chiller
- 6 Hourly load factor of absorption chiller

The equipment efficiency is determined as shown in Table 2. The generating efficiency of commercial power and waste heat recovery efficiency, waste heat recovery boiler efficiency, absorption chiller COP, backup boiler efficiency, and heat pump chiller COP are the constant values. As shown in Figure 13, the power generation efficiency of a generator depends on the rated output and load factor.

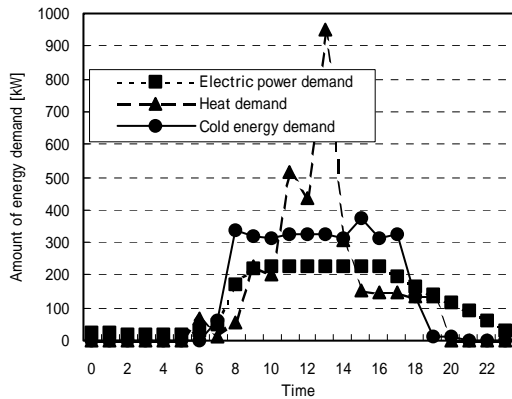


Figure 11. Assumed Energy Demand

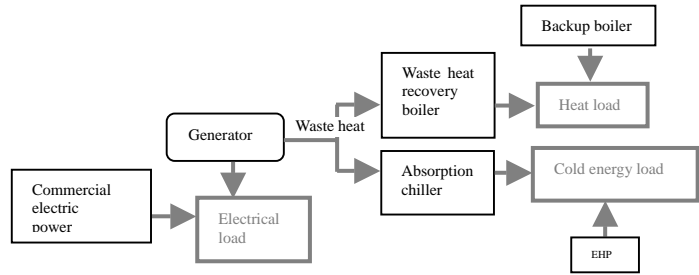


Figure 12. Examined CGS configuration to evaluate the algorithm performance

Table 2. Equipment Efficiency

Equipment	Efficiency
Power generator	Depends on rated output and load factor ( see Figure 13 )
Commercial electric power	0.35
Waste heat recovery equipment	0.9
Waste heat recovery boiler	1.0 (0 if load factor is 25% or less)
Absorption chiller	1.0 (in COP) (0 if load factor is 25% or less)
Backup boiler	1.0
Heat pump chiller	3.0 (in COP)

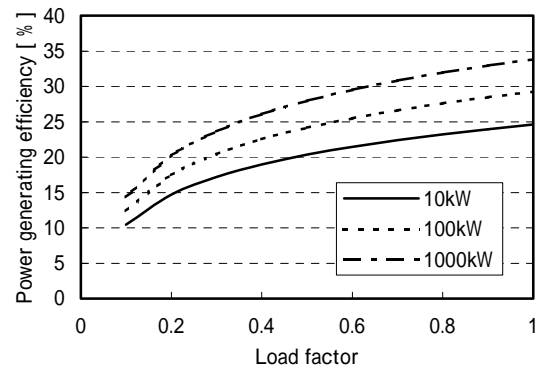


Figure 13. Power Generating Efficiency

## (2) Results of optimization

As a result of optimization under the condition of the primary energy consumption minimum, it was found automatically the system consists of a generator rated output of 147kW, a waste heat recovery boiler rated output of 196kW, and an absorption chiller rated output of 185kW (See Figure 12).

The generator hourly output; namely, the product of the searched load factor and the rated output, and the amount of commercial power used; namely, the generator output subtracted from the electric power demand are shown in Figure 14. The waste heat

recovery boiler hourly output; namely, divided into an effective component and an ineffective component in excess of the heat demand and the backup boiler output; namely the waste heat recovery boiler output subtracted from the heat demand are shown in Figure 15. The absorption chiller hourly output, which is divided into an effective component and an ineffective component in excess of the cold energy demand, and the heat pump chiller (EHP) output are shown in Figure 16. As these figures indicate, it is clear that EDA could find operation patterns that would operate the generator when there was a heat demand and/or a cold energy demand, that is, when waste heat is used.

Figure 17 shows the relationship between the number of generations and the cogeneration system efficiency of better individual about classical GA and EDA, that is to say, the comparison of reasoning performance of GA and EDA. As this figure shows, it is also revealed that the optimum system cannot be found with GA, which is a conventional optimization algorithm, and that optimum system configuration and operating pattern can be obtained more efficiently and with less calculation work with EDA.

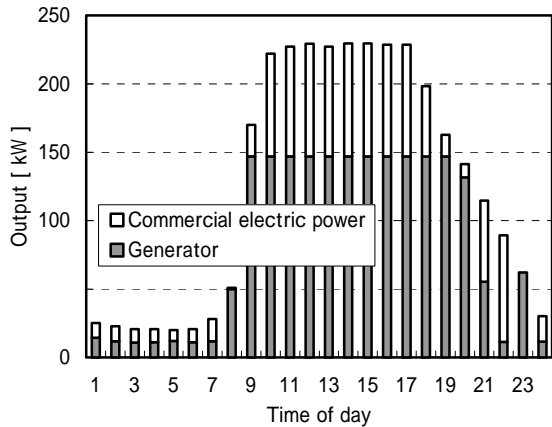


Figure 14. Optimum Operation Pattern (Electric Power)

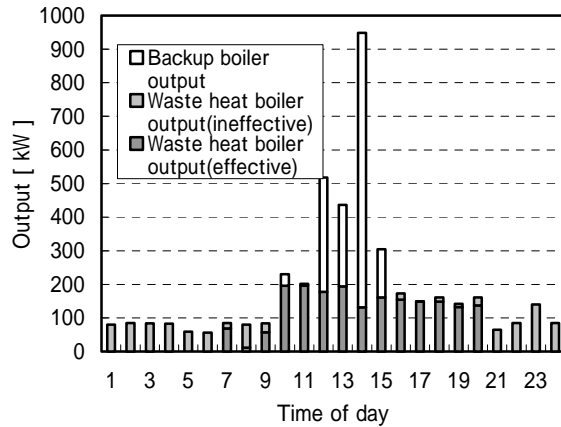


Figure 15. Optimum Operation Pattern (Heat)

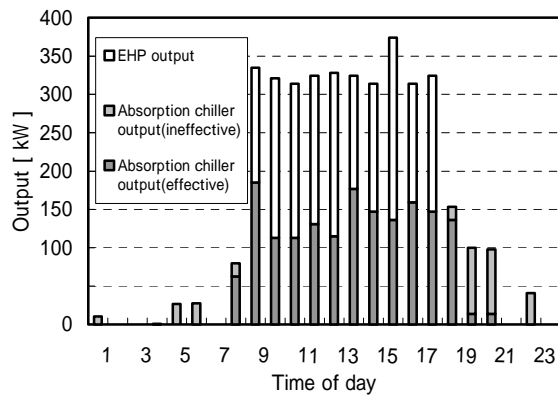


Figure 16. Optimum Operation Pattern (Cold Energy)

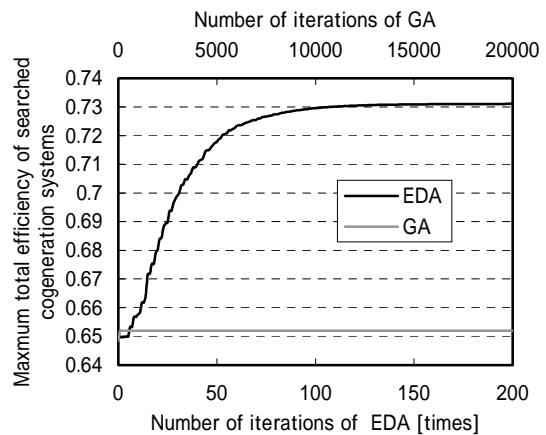


Figure 17. Comparison of Reasoning Performance of EDA and GA

## Conclusion

- An evaluation function was developed that is appropriate for computerized automatic optimization, and that enables more accurate CGS energy calculations.
- EDA was applied, adopting the principle component analysis, into CGS optimization. As a result, excellent optimal system searching ability was shown with the EDA, whereas it was impossible to optimize CGS with a simple GA.

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